CONSTITUTIVE MODELS FOR TRANSVERSELY ISOTROPIC FIBER PREFORM IN COMPOSITE MANUFACTURING

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Keywords: *Transversely isotropic preform, Dual scale flow, Process modeling.s*

Introduction

The present contribution is an attempt to model a prepreg that is assumed to have approximately straight and parallel fibers which in an unloaded state has a transversely isotropic symmetry about the fiber axis $\boldsymbol{n}$. It is further assumed that the prepreg behaves elastic under a purely volumetric deformation, as well as under axial stretching in the fiber direction. A two-scale flow highly coupled to the fiber bed deformation is also being modelled using poromechanics. The framework comprises a nonlinear compressible fiber network saturated with incompressible fluid phase.

Hyperelasto-hypoplasticity of a transversely isotropic fibre bed

In the present development we consider assemblies of parallel fibres for the reinforcement of composite materials. These are strongly anisotropic and display a complex variety of responses in different directions, ranging from simple elasticity to granular flow depending on the orientation and mode of deformation. To describe this behaviour, in a single consistent theory, we combine a hyperelastic non-slip response based on a strain energy potential, for the volumetric mode and extension in the fibre direction, with plastic frictional slip modes based on a stress energy potential, for the transverse and longitudinal shear modes.

Constitutive structure

The material under consideration is a bed of dry fibres that are approximately straight and parallel. In its unloaded state, the material is assumed to have transversely isotropic symmetry about the fibre axis $\boldsymbol{n}$. The material is further assumed to be elastic under purely volumetric deformation, as well as under axial stretching in the fibre direction. Any shear deformation, however, will be governed by frictional interaction of a granular solids nature. To allow for these fundamentally different responses, we divide the Cauchy stress $\boldsymbol{\sigma}$ into two parts: a conservative, hyperelastic, stress $\boldsymbol{\sigma}_0$ and a non-conservative stress $\boldsymbol{\sigma}^*$. The conservative stress is due to transverse compression and longitudinal extension of the fibre bed, and is therefore linked directly to the volume fraction ϕ and fibre extension λ,

$$\boldsymbol{\sigma}_0 = \lambda(\phi, \lambda, \boldsymbol{n}). \tag{1}$$

The non-conservative stress is due to frictional shear forces within the fibre bed, and will be described in the spirit of granular solids, by means of a differential evolution law:

$$\dot{\boldsymbol{\sigma}}_* = \mathcal{G}(\phi, \boldsymbol{\sigma}_*, \boldsymbol{n}, \boldsymbol{l}) + \boldsymbol{\omega}\boldsymbol{\sigma}_* - \boldsymbol{\sigma}_*\boldsymbol{\omega}, \tag{2}$$

where $\boldsymbol{l}$ is the velocity gradient, with symmetric and skew symmetric parts $\boldsymbol{d}$ and $\mathbf{w}$. The material spin, $\boldsymbol{\omega}$, should naturally follow the rotation of the fiber axis such that $\boldsymbol{\omega}\boldsymbol{n} = \dot{\boldsymbol{n}}$, and it is assumed to co-rotate objectively about the fiber axis by $\boldsymbol{n} \cdot \boldsymbol{e} : \boldsymbol{\omega} = \boldsymbol{n} \cdot \boldsymbol{e} : \mathbf{w}$. The spin that satisfies these requirements is

$$\boldsymbol{\omega} = \mathbf{w} + \boldsymbol{dnn} - \boldsymbol{ndn}. \tag{3}$$

The non-conservative stress is the shear stress along and across $\boldsymbol{n}$, thus

$$\boldsymbol{\sigma}_* : \boldsymbol{nn} = \mathbf{0}, \qquad \boldsymbol{\sigma}_* : \boldsymbol{I} = \mathbf{0}. \tag{4}$$

Results

As a starting point developed model was evaluated against phenomenological behavior. Thus the numerical integration of governing equations was performed using a quadratic algorithm. The quadratic algorithm was chosen in order to stabilise oscillating stress response at steady flow due to the chosen simple Jaumann-type co-rotational axial spin. More sophisticated spins, such as the logarithmic spin, can of course be used if required. The initial verification of the

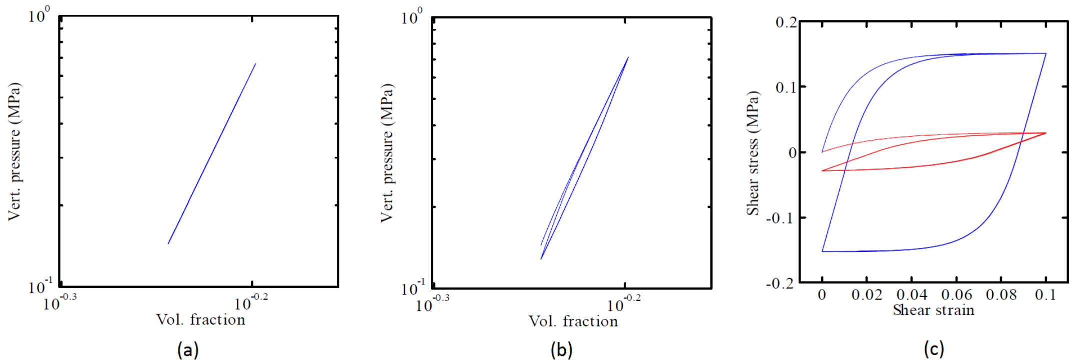

Figure 1: *(a) Biaxial transverse compression, 10% volumetric, two cycles starting at $\phi_0 = 0.57$, (b) Uniaxial compression, 10% volumetric, two cycles starting at $\phi_0 = 0.57$, (c) Two cycles of 10% longitudinal and transverse shear strain.*

implementation is presented in Figures 1a-c. Figure 1a plots the transverse pressure $-\sigma_{33}$ vs volume fraction ϕ in biaxial cyclic compression. Being hyperelastic, this response is completely reversible with no sign of hysteresis. Figure 1b shows same thing for uniaxial compression. Two cycles are shown for a compression strain of 10%, starting at $\phi_0 = 0.57$, the process is stationary already after two cycles, so all further cycles coincide with the second cycle. The hysteresis is due to the shear component involved in the uniaxial compression mode. Figure 1c plots the shear stress response in longitudinal and transverse shear. Two cycles are shown for a shear strain of 10% at $\phi_0 = 0.6$. Again the process is already stationary after two cycles.

References

References

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